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Set Theory and Fuzzy Sets: Their Relationship to Natural Language

An Interview with George Lakoff[1]

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Background: A recurring argument of scholars who advocate set theory and fuzzy sets for social science is that this framework is valuable and appropriate in part because it reflects the structure of meaning in natural language.[2] George Lakoff has written extensively on these topics and is cited by these scholars as an authority. In this interview, Lakoff synthesizes a large body of research in linguistics and cognitive science, which contends that natural language is not set-theoretic in structure. He also explores Lotfi Zadeh's fuzzy logic, emphasizing both its creative applications in engineering and its poor fit with most features of natural language. Finally, Lakoff discusses the basic contrast between Zadeh's fuzzy logic and Charles Ragin's fuzzy-set scoring. Lakoff emphasizes that he is not in a position to judge the substantive contribution of Ragin's method. However, it does not rely on an empirically adequate account of natural language; and because the scoring is based on fixed numerical values, rather than fuzzy distributions, Ragin's scoring does not qualify as a fuzzy method.

Q: Is natural language set-theoretic?

A: Standard set theory—I will discuss fuzzy sets later on—does not capture the structure of natural language. Categorization is one of the primary means by which humans use natural language to understand the world. The set-theoretic view is based on what we call the classical theory of categorization. This theory posits that we categorize objects or experiences in terms of inherent properties that are necessary and/or sufficient for category membership. In standard set theory, objects and experiences are understood as either inside or outside a specific category. Anything that has a given combination of inherent properties is inside the category, and anything that does not have these properties is outside the category. In the classical theory, there are no degrees of category membership: It's *in* or *out*.

However, this set theoretic concept of categorization does not correspond to the way people categorize objects and experiences using natural language. As Rosch (1975, 1977) has found, we instead categorize in terms of prototypes and family resemblances. Unlike set theory, the theory of prototypical categorization, as extended in my book *Women, Fire, and Dangerous Things* (Lakoff 1987; hereafter *WFDT*), is sufficiently flexible to capture the category structure of natural language. For example, the prototypical chair has a back, seat, four legs,

[1] This interview was conducted in December 2013. Lakoff subsequently revised and amended the text and provided the bibliography.

[2] For references, see footnote 10 in David Collier's Introduction to this symposium.

and usually armrests; it is used for sitting. But we also have nonprototypical chairs—wheelchairs, hanging chairs, and dentists' chairs. We understand the nonprototypical cases as chairs by virtue of their relationship to the prototype. In this way, category structures defined by such prototypes map directly onto the way we conceptualize and describe objects using natural language.

The idea of family resemblances becomes crucial here. We understand nonprototypical chairs as being chairs because they bear a *family resemblance* to the prototype. Family resemblances are not linearly ordered; one thing can bear a resemblance to another in various ways along various dimensions. The theory of radial categories in *WFDT* provides an account of what constitutes a family resemblance. Real radial categories are more complex, incorporating metaphoric and metonymic relations. The radial category structure defines not only what is "sufficiently close," but also the nature of the difference between prototype and object.

Unlike standard set-theoretic categorization, which does not allow us to readily categorize objects or ideas that stretch the limits of a set, prototypes and family resemblances can be systematically extended to define relationships between categories. Modifiers, which I have described as *hedges* (Lakoff 1973), include expressions such as "strictly speaking," "loosely speaking," and "par excellence." The hedges change the category boundaries in ways dependent on context and reflect the structure of prototypical and nonprototypical members. *Strictly speaking* picks out the central examples. *Loosely speaking* somewhat extends the prototype boundaries and eliminates the prototypical examples. *Par excellence* again redraws the category boundaries to include only the best prototypical examples. For example, a robin is a bird *par excellence*, while chickens and ostriches are not.

There are also types of prototypes with different properties: social stereotypes, typical cases, paragons, nightmare cases, salient exemplars, generators, and so on. Standard set theory is too rigid to capture the relationships between categories and families of categories.

We have learned a lot about the structure of family resemblances—specifically, how we pick out things that are similar in certain respects, and different in others. The theory of conceptual metaphor—which is now grounded in new work on the neural theory of thought and language and experimental research on embodied cognition—has been a major advance in understanding real cognitive structure. And of course, metaphor is not just a poetic or literary device, but a basic feature of largely unconscious everyday thought and language. Conceptual metaphors are frame-to-frame mappings that allow a source frame to project content onto a target frame, thus greatly enriching our means of conceptualization. Conceptual metaphors either have a direct bodily grounding or are decomposable into more primitive metaphors that have a bodily grounding. The system of embodied conceptual metaphor is the broad super-structure of our system of concepts. Conceptual frames and metaphors form networks called "cascades," which are used in characterizing the content of categories.

Set theory has none of this real cognitive apparatus.

Q: Say more about the contribution of Rosch.

A: Rosch was a pioneer in breaking with classical categorization. Her experiments in the mid-1970s strongly support the idea that human categorization is organized around prototypes and family resemblances. As an undergraduate at Reed College, she wrote her honors thesis on Wittgenstein and engaged with his concept of family resemblances—the idea that objects in a given category do not necessarily have common properties, but resemble each other like family members who have different combinations of shared features. While this famous idea is only briefly articulated in Wittgenstein's writings, it became crucial for Rosch's research.

Later, as a graduate student at Harvard, Rosch worked closely with Roger Brown, the author of "How Shall a Thing Be Called?" This led to her groundbreaking work on basic-level categories. It was previously thought that categories were simply hierarchical, and that lower-level categories were just special (less general) cases of higher categories. Thus, in this general-to-specific hierarchy, sports cars were seen as special cases of cars, which were seen as special cases of vehicles, while rocking chairs were seen as special cases of chairs and chairs as special cases of furniture.

By contrast, Rosch showed that these categories in the middle—cars and chairs—have special properties. They are defined by a confluence of motor programs, mental images, and gestalt perception. They also tend to be learned first and often have the shortest names.

We now have a neural explanation for this confluence of properties. Mirror neuron system research shows common circuitry linking motor programs and gestalt perception, and Martha Farah's (1989) research demonstrates that mental images use the same circuitry as the visual system. That explains why motor programs, gestalt perception, and mental images fit together in defining basic-level categories. There is nothing in set theory that can deal with those phenomena. Most important, Rosch showed that basic-level categorization is embodied. Set theory, of course, is disembodied. The Brown-Rosch research was confirmed in the work of Berkeley linguistic anthropologist Brent Berlin, who showed that the level of the genus in biology has the properties of the basic level.

In short, research on both prototypes and basic-level categories shows that the real capacities of natural language do not have the structure of set theory and go far beyond what classical set theory can do.

Let me be more specific about Rosch's contribution. Her remarkable work revolutionized the empirical study of categorization. She conducted path-breaking experiments on the Dani people of Papua New Guinea in the early 1970s, and performed further experiments at Berkeley. Rosch's New Guinea experiment involved teaching the Dani a series of made-up focal and nonfocal color terms. She found that they remembered focal color terms—which represent more basic colors, such as red or green—far more easily than nonfocal color terms, which represent complex colors, such as red-orange or pink. In later work she explored the conceptual structure of categorization, showing that people more readily identify prototypical cases as members of a category and have a quicker response time to questions about prototypical versus non-prototypical cases.

In her Berkeley experiments, Rosch asked subjects to rate a series of terms according to how well they exemplify a certain category. For the category of weapon, for example, she derived from the responses a scale that ranked sixty objects in terms of their centrality. Gun ranked at the top. Bayonet, arrow, fists, and words were successively further from the prototype, with dozens of other objects ranked in between. Based on these and similar results for many other categories, Rosch found that respondents recognized a spectrum of similarity, with an ordered sequence of representativeness in relation to the prototype. Using these innovative methods, she established that people categorize a given object or experience by comparing it with the object or experience—the prototype—they think best represents a given category. As I will explain later, these initial discoveries were a key step—though only an initial step—in developing prototype theory.

In sum, Rosch's work is indeed fundamental to the empirical research on which our understanding of categorization rests. In my 1980 book with Johnson, *Metaphors We Live By*, she is a central point of reference for the argument that we do not categorize in set theoretic terms. In my 1987 book, *WFDT*, I survey her work in much more depth to support the argument that set theory does not reflect categorization in natural language, and my book on mathematics (Lakoff and Núñez 2000) likewise underscores these themes.

Stepping back from Rosch's work, we can say that different set theories place distinct constraints on what they can say about any given domain. We find technical subjects for which set theory is useful; certain types of computer databases were developed to fit classical set theory. Some programming languages, such as HTML, required new and very different set theories—developed in part at the International Computer Science Institute (ICSI) at Berkeley. But if the topic of concern is natural language and human conceptual systems, all set theories are going to fail.

Q: Let us now focus on fuzzy sets and fuzzy logic, which are central to these discussions of set theory in social science. Would you give us your views on Lotfi Zadeh and fuzzy-set theory?

A: I would first point out that Zadeh (1965, 1972) initially developed what he called fuzzy-set theory. I added to this theory by introducing my idea of hedges and of different fuzzy logics. Zadeh built on the work on hedges and created what we now think of as his "fuzzy logic."[3] I will use that term to refer to his contribution.

For me, Zadeh is an admirable scholar. The application of his ideas in engineering is remarkable. Zadeh's fuzzy logic was developed into algorithms and chips used in engineering contexts like rice cookers, vacuum cleaners, washing machines, refrigerators, and especially anti-lock brakes (in the brakes of my car). Zadeh and others have developed fuzzy logic control systems, on which there is a large technical literature.[4] Such systems are useful in devices with ongoing *multiple* linear inputs that require smoothly functioning, *single* linear outputs.

The important contribution of fuzzy logic becomes clearer with an example. When someone applies pressure to the brakes in their car, there is an infinite array of values that the amount of pressure can take. Yet the amount of pressure we apply does not vary in continuous gradations, but rather is closer to a step function. These values or steps can be operationalized using hedges—*moderate* pressure, *strong* pressure, and so on—and each hedge can be graphed with what I have called a "Zadeh function."

Fuzzy logic is more useful than, say, linear scales for capturing this process of braking. Because the functions are anchored in hedge terms, they have clearly defined substantive meanings. Using Zadeh's theory, engineers can thus translate the amount of pressure drivers apply to the brakes into functions that can be visualized as the pressure transitions across a spectrum *light* to *moderate* to *strong*.

Fuzzy logic allows engineers to work with increased precision, and it represented an impressive leap for engineering. Zadeh deserves all the acclaim he has received in the engineering world, especially in Japan.

The question for social scientists is whether any real social or political phenomena work like rice cookers or washing machines, and whether fuzzy logic distorts reality and fails in domains that do not work this way.

Q. You and Zadeh had a dialogue over the relationship between fuzzy logic and linguistic hedges. Would you describe that?

A. We started exchanging ideas in the early 1970s. I had previously made an extensive list of linguistic hedges that serve to modify categories. Most of them were complex natural language cases which did not fit Zadeh's fuzzy logic. A small number, however, fit ordered linear scales—for example, *extremely*, *very*, *pretty*, *sort of*, *not very*, *not at all*. Yet these still did not fit Zadeh's original version of fuzzy logic for a simple reason: the original version placed the values for set membership not just on an ordered scale, but on an infinitely-valued, continuous scale between zero and one. That does not correspond to natural language.

Zadeh understood the problem when I described it to him, and he suggested an ingenious solution in his 1972 article, "A Set-Theoretic Interpretation of Linguistic Hedges." Here he developed a version of fuzzy logic that drew on my hedges paper (later published as Lakoff 1973). He defined a group of mathematical functions taking the real numbers from his original fuzzy logic as input. Each was a Gaussian curve peaking at values that approximated ideas like those expressed by linear hedges such as *very*, *sort of*, *not very*, and so on. These curves incorporated the idea of imprecise, fuzzy gradations around each hedge. The output of these functions defined a new kind of fuzzy logic with a small number of linearly ordered values instead of a continuous spectrum of values. In my 1973 paper, I called these "Zadeh functions." Zadeh (1972) called the resulting set-theoretic logic a "hedge logic," a term that continues to be used (van der Waart van Gulik 2009).

[3] See, for example, Zadeh 1995: 271; also Zadeh 1994.

[4] See the bibliography provided in even as ordinary a source as the Wikipedia entry on Fuzzy Control System.

As his new theory of fuzzy logic evolved for engineering applications, Zadeh simplified the curves to linear triangular and trapezoidal functions. In the triangular functions, the peaks of curves are replaced by points, and the curves leading up to the peaks are replaced by the straight sides of the triangle. In the trapezoidal functions, the peaks of the curves are flattened to encompass a range of values, and the curves going up to the peaks are represented by the straight sides of the trapezoid. That makes engineering computations easier.

I should reiterate that the linear hedges used in Zadeh's hedge logic are a minority of the hedges in English. Hedges like *basically*, *essentially*, *regular*, *technically*, *so-called* and many others cannot be handled by fuzzy logic, and Zadeh has never claimed they could be. Moreover, many modifiers are nonlinear and their compositions with nouns cannot be handled by the compositional functions of fuzzy logic. Well-known cases cited in *WFDT* that require frame semantics include *electrical engineer, social scientist*, *mere child*, *fake gun*, *happy coincidence*, *past president*, and many more.

Q: You are suggesting that fuzzy logic does not reflect the structure of meaning in natural language. Would you spell this out?

A: Fuzzy logic does not characterize most of the human conceptual system as it is found in natural language. It cannot characterize frame semantics, conceptual metaphors, conceptual metonymies, conceptual blends, modalities, basic level concepts, radial categories, most hedges, most conceptual composition, and so on. It especially cannot handle the broad range of contested concepts, especially important ones like freedom and democracy that depend on conceptual metaphor, morally-based frames, and radial categories. It cannot account for the experimental results in embodied cognition research.

My 1973 hedges article is sometimes cited as if it were an endorsement of fuzzy logic, but it in fact discusses many limitations. Let me spell out what I said then—and the context was of course my admiration for Zadeh and my collaboration with him.

I noted in the 1973 hedges piece that fuzzy concepts have had a bad press among logicians, and that these concepts merited serious formal study. I tried to suggest how this formal study should be focused.

It is exciting to think back to 1973, when this article was published. What can be called the Berkeley Revolution in Cognitive Science had only begun. Rosch had just started her path-breaking empirical work, and I refer to that in my article. I had not yet developed the idea of radial categories, which later drew together her work and the emerging literature on frame semantics. But elements of these ideas were present.

I identified different types of hedges, and some are amenable to the linear treatment provided by fuzzy logic. Fuzzy logic is linear in the sense that elements are consistently ordered along a line. Many other hedges definitely are not, and the more I developed these ideas, the more I realized that most hedges modify the central category in diverse ways that are definitely non-linear. Zadeh ingeniously identified a few hedges that were very successfully modelled in his engineering applications. I applaud this. But as an overall characterization of natural language, fuzzy logic fails.

Q: Zadeh's 1982 article "A Note on Prototype Theory and Fuzzy Sets" sought to show that fuzzy logic can accommodate the idea of prototypes. Did he succeed?

A: Zadeh's article fails to make the case. First of all, Zadeh only considers the initial version of prototype theory, in which Rosch shows that within categories, there can be a finite hierarchy from examples that are best, good, less good, and so on. Zadeh says this hierarchy shows fuzzy logic is compatible with natural language. Yet even for this initial version of prototype theory, fuzzy logic is inadequate. The initial version is centered on the idea of closeness based on properties related by family resemblances; by contrast, fuzzy logic takes into account neither properties nor family resemblances and is based on a continuous, infinitely-valued linear scale.

Moreover, the fully developed theory of prototypes is more complex than the linear conceptualization suggested by the initial version. It encompasses the use of prototypes to stand for the category as a whole (i.e., metonymy) with respect to some form of reasoning. For instance: (a) Best example prototypes function as defaults where only the category is mentioned. Thus, if you say "There's a bird on the porch," you will most likely have in mind a small songbird, not a duck that could have flown in from a nearby lake, nor an ostrich from an ostrich farm, and definitely not a pelican that might have strayed from the ocean. (b) Typical case prototypes are used for drawing inferences. (c) Reference point prototypes are used to provide a standard in reasoning. (d) Salient example prototypes are used for judging probability. (e) Ideal prototypes are used for making value judgments. Fuzzy set theory does not have any of these properties.

Another element in prototype theory that is not accommodated by fuzzy logic is the idea of radial categories, which capture how cases branch out in many directions from the central members. For example, there are cluster categories defined by a cluster of frames, with modifiers that only pick out one of the frames. The category *mother* is defined by four frames—for birth, genetics, nurturance, and marriage. But stepmother eliminates marriage, and birth mother picks out birth but not marriage, and genetic mother picks out genetics, but not necessarily birth, and so on.

The linear ordering of fuzzy logic certainly does not reflect this pattern. In this and many other ways, by the early 1980s studies of human categorization had left fuzzy logic far behind.

In sum, it is valuable that Zadeh recognized the importance of prototype theory. But he failed to connect it with fuzzy logic.

Q: Do the inadequacies of fuzzy logic for natural language lead to inadequacies for applications to political and social analysis?

A: Yes, definitely. Good examples would be concepts of freedom and democracy. In my 2007 book *Whose Freedom?* I analyzed this contested concept by extending and refining W. B. Gallie's (1956) theory of contested concepts, an outstanding example of work by a political theorist that captured important facets of human conceptual structure. Inspired by George W. Bush's second inaugural address, where Bush used the words *freedom*, *free*, and *liberty* 49 times in 20 minutes, I undertook to characterize (1) the shared conception of freedom used by both progressives and conservatives, and then (2) the contested extensions of this shared conception, which differ widely between progressives and conservatives. The differences are huge, and the book covers how they apply to a wide range of social and political contexts, from economic markets, to education, religion, foreign policy, human rights, and gender issues.

These vital distinctions for our politics, and the politics of many countries, cannot be approached in any linear fashion in relation to a general concept of freedom, which is what fuzzy logic would require. The same is true of democracy, as Elisabeth Wehling and I pointed out in *The Little Blue Book* (Lakoff and Wehling 2012). Wehling (2013) subsequently—in collaboration with social psychologists—pursued this line of inquiry further using survey and experimental methods. She confirmed the contested conservative versus progressive extensions of the shared core I found in *Moral Politics* (2002) and extended in *The Political Mind* (2008). None of this research fits fuzzy logic.

Q. How would you compare Zadeh's fuzzy logic with Charles Ragin's (2000, 2008) method of scoring fuzzy sets? To avoid confusion, we can refer to these as "fuzzy logic" and "fuzzy-set scoring."

A. They are very different. Zadeh arrayed complex functions on a linear scale to approximate the fuzziness of hedges like *very*, *pretty much*, *sort of*, and *not much*. Each hedge is represented by a complex function. The overall scale is indeed linear, in that the hedges have a well-defined linear order. The input to the functions is the set of real numbers from zero to one. However, the core idea for Zadeh is that the meaning of each specific hedge is fuzzy.

By contrast, with Ragin's method of fuzzy-set scoring, the entire approach is linear. Based on a completely different, non-fuzzy approach, full membership in the overall category is represented by a fixed numerical value, and each hedge also has a fixed numerical value. It is not fuzzy.

Let's set aside for now my argument that most hedges cannot in fact be arrayed on a linear scale. Zadeh's hedge logic is nonetheless a worthy attempt to capture the linear ordering of some fuzzy hedges, and he thereby did something important. We noted the example of pressure: *light pressure*, *moderate pressure*, *strong pressure*, *intense pressure*. In his system, the overall ordering is indeed linear, and the use of fuzzy logic to represent these hedges is interesting, subtle, and valuable in engineering.

By contrast, I am skeptical that Ragin's fuzzy-set scoring can tell us much about the conceptual understanding of the real world that is contained in natural language. I do not have any serious knowledge of the substantive contribution to social science, so I will only comment on the conceptual part.

The examples I have seen of fuzzy-set scoring in social science are, to reiterate, quite different from Zadeh. The subtlety of fuzzy logic is gone. The subtlety of hedge logic, which captures the fuzziness of each step (hedge) on the linear scale, is gone. It seems to have nothing to do with the complex meaning of hedges, or with anything else in natural language.

Instead, in fuzzy-set scoring the analyst constructs a scale with—for example—three, or sometimes five, values. Sometimes there are more values. The values are evenly spaced, fixed numbers that are arrayed linearly: 1.0, 0.5, 0.0; or if there are five values, 1.0, 0.75, 0.5, 0.25, 0.0. This is completely different from what Zadeh did with fuzzy logic. With fuzzy-set scoring, full membership in the category is scored as 1.0, non-membership is scored as 0.0, and the crossover (tipping) point as 0.5. Leading examples of the overall categories analyzed (Ragin 2000, 2008) include *rich countries*, *Protestants*, *major urban areas*, and *developed countries*—obviously important topics for social scientists.

In fuzzy-set scoring, analysts assign values based on their own interpretations, often combined with a mapping from standard linear measures. For example (Ragin 2000:158), they take the measure of GNP per capita as the basis for assigning membership in the category of rich countries. $18,000 to $30,000 per capita is assigned to *clearly rich* (score=1.0), $8,001 to $17,999 to *more or less rich* (0.75), $8,000 is *in between* (0.5), $2,000–$7,999 is *more or less not rich* (0.25), and $100 to $1,999 is *clearly not rich* (0.0). These are hedges, but they are represented with these fixed numerical values, rather than fuzzy functions. Except for the guidance from conventional linear indicators, the principle behind choosing these values is unclear, and I find the discussions of external anchors for this assignment unconvincing. My goal (1973) in analyzing hedges in natural language was to explore their meaning and fuzziness, and Zadeh attempted to capture the idea that their meaning is fuzzy. I don't find that in fuzzy-set scoring.

I have my own misgivings about economic indicators, and I worry about what they hide. But I would prefer to know the GNP per capita of a country, rather than be told that it has a fuzzy-set score of 0.5. This score indicates that it is exactly halfway between being a full member and a full non-member in the set of rich countries, but I don't know what it means conceptually or empirically to be in this set.

In sum, fuzzy-set scoring seems to rely on a rigid, fixed threshold for full membership and for the intermediate values. With Zadeh, these thresholds are, by contrast, fuzzy. In terms of capturing meaning in natural language, with fuzzy-set scoring I don't see the gain over conventional indicators—whatever their limitations. And to reiterate, I do not view fuzzy-set scoring as actually being a fuzzy method.

Q: Do you have concluding comments about these applications of set theory and fuzzy logic?

A: To reiterate, a common justification offered by texts on set theory in social science is that fuzzy set theory and fuzzy logic capture meaning in natural language.

That is simply wrong. It is not supported by the empirical literature on conceptual systems in natural language. Given this mistaken justification, it is hard for me to understand how any social scientist can take set theory and fuzzy logic seriously.

The goal of the fuzzy logic approach seems to be to represent complexity, but for most phenomena, this approach cannot and does not succeed. While fuzzy logic does have impressive applications in engineering, it fails to address the complexity of data routinely examined in the social sciences. Real political and social phenomena do not fit the constraints of fuzzy logic control systems.

One might be tempted to dismiss the application of Zadeh's fuzzy logic to social and political science as misguided. That would be a great mistake and would fail to honor Zadeh's contribution. The important point is that the technical tools of fuzzy logic *define* the data it can fit. The danger is that the technology can distort what should count as real social science data. This danger is certainly also present in many "big data" statistical methods, which define the relevant data as what the technology can do.

The real issues here are empirical: (1) Does the model fit reality? (2) Does it fit the way we conceptualize reality? Personally, I doubt that Ragin's fuzzy-set theory will work in either case. Again, as a cognitive scientist and linguist, I can only judge how set-theoretical tools fit human thought and language. I am not in a position to judge the empirical utility of Ragin's model from other scientific perspectives.

However, I believe that social scientists would do well to look for alterative tools, ones that reflect human conceptual systems and are appropriate to the phenomena that need to be studied.

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